Dynamic fluctuations of dipolar semiflexible filaments

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On the basis of the model of a flexible magnetic filament, the characteristics of their thermal fluctuations are considered. The crossover of the time dependence of the mean quadratic displacement from $t^{3/4}$ to $t^{1/2}$ at the magnetic field increase is found. Two characteristic mechanisms of the magnetization relaxation time distribution—straightening of the thermal undulations and excitation of the bending modes of the free ends under the action of an ac magnetic field—are described. In both cases, the characteristic scaling law $\omega^{-3/4}$ of the magnetic susceptibility in a high-frequency range is found.

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I. INTRODUCTION

The dynamics of different semiflexible filaments is responsible for viscoelastic properties of their networks [1]. Scaling laws for the dependence of their dynamic susceptibility on the frequency of external perturbation and characteristics of thermal fluctuations of the filaments are known [2,3]. The model of a dipolar semiflexible filament proposed recently [4,5] shows that the characteristic exponents for the dynamic susceptibility depend on the magnetic field strength [6,7]. Here this dependence is studied in detail.

The scaling laws of the dynamic susceptibility are important, for example for the understanding of the properties of the magnetic colloids containing chains of the single domain ferromagnetic nanoparticles [8,9]. The persistence length of these chains can exceed the diameter of the particles more than 10 times [5]. The samples containing chains of the magnetic particles show rather unusual dispersion of the magnetic susceptibility [8]. The chains of superparamagnetic particles also play an important role in the behavior of the magnetorheological suspensions [10]. It was shown recently [11] that in the range of the parameters where magnetic interaction between the particles is attractive, the behavior of the chains of free magnetic particles can be described by the model of a flexible magnetic filament [4,5]. Thus the investigation of the dynamic properties of the dipolar semiflexible magnetic filaments is important for this class of materials too. Last but not least, the magnetotactic bacteria should be mentioned [12]. An investigation of dynamic susceptibility of these micro-organisms can deliver important information on the organization of the magnetosomes of the bacteria [13]. The rich and unusual behavior of these bacteria under the action of the rotating field was investigated recently [14].

Here in the Sec. II, the characteristics of the thermal fluctuations of the chains of dipolar particles are studied. It is shown that in dependence on the observation time, the crossover of the mean quadratic displacement of the chain from the $t^{3/4}$ law characteristic to the semiflexible polymers [15] to the $t^{1/2}$ characteristic to the flexible magnetic filaments takes place. Since the magnetic filaments are fluctuating, their shape deviates from the straight rod. The magnetic field component along the filament suppresses these fluctuations. Taking them as small in Sec. III, the time dependence of the long filament magnetization due to the thermal fluctuation suppresion by the applied ac magnetic field is considered. In this case, the contribution of the ends of the filament is not important and the frequency dependence of the susceptibility (dispersion) is due to the described effect of the straightening of thermal fluctuations. The dispersion of the magnetic susceptibility of the sample of randomly orientated chains of magnetic particles with a finite length in dependence on the frequency of a small ac magnetic field is considered in Sec. IV. The characteristic scaling laws of the dynamic susceptibility are obtained and compared with the available experimental data [8].

II. THERMAL FLUCTUATIONS OF DIPOLAR SEMIFLEXIBLE MAGNETIC FILAMENTS

Let us consider the magnetic filaments with magnetization M per unit of its length. The shape of the filament is characterized by its tangent orientation angle ϑ with respect to the x axis, which is in the direction of the applied field H (see Fig. 1). In the case of the small deformations of the filament, the effects of its tension are negligible. In this case, the equa-

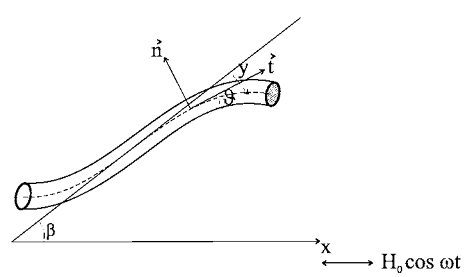

FIG. 1. A picture of a magnetic filament

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tion for a small perturbation of the tangent angle $\tilde{\vartheta}$ of the filament, oriented at angle β with respect to the applied field, from the value β is [4,5] (hereafter, l is the contour length of the filament, and the tilde is omitted)

$$\partial_t \zeta \vartheta = - \partial_{llll} C \vartheta + \partial_{ll}(F_n). \tag{1}$$

Here $\zeta = 4\pi\eta/[\ln(L/a)+c]$ is the hydrodynamic friction coefficient per unit length of the filament (L is the length of the filament, a is the radius of its circular cross section, and c is constant with the order of magnitude 1) and C is the curvature elasticity constant. The normal force F_n due to the action of the torque on the filament from the applied field $T_0 = -MH \sin(\beta+\vartheta)$ up to the first-order terms in tangent angle perturbation is ($H_\parallel = H \cos \beta$)

$$F_n = - T_0 \cong MH \sin \beta + MH_\parallel \vartheta. \tag{2}$$

As a result, the equation for the small perturbation of the tangent angle is as follows:

$$\partial_t \zeta \vartheta = - \partial_{llll} C \vartheta + MH_\parallel \partial_{ll} \vartheta. \tag{3}$$

In the case of the long chain, we can represent ϑ by its Fourier series ($k = \pm 2\pi n/L, n = 0, 1, 2, \ldots$)

$$\vartheta(l,t) = \frac{1}{L} \sum_n \vartheta_k(t) \exp(ikl).$$

For the Fourier amplitudes, the following equation is valid:

$$\zeta \frac{d\vartheta_k}{dt} = - Ck^4 \vartheta_k - MH_\parallel k^2 \vartheta_k + f_k^\vartheta, \tag{4}$$

where the term f_k^ϑ representing the effect of thermal noise is accounted. The statistical properties of thermal noise are determined by the fluctuation-dissipation theorem [16]

$$\langle f_k^\vartheta(t') f_{k'}^\vartheta(t'') \rangle = 2\zeta k_B T L k^2 \delta(t'-t'') \delta_{k,-k'}.$$

The Fourier amplitudes of the displacement of the filament y from its straight configuration are found from the relation $\partial_l y = \vartheta$. The solution of the Cauchy problem with initial condition $\vartheta_k(0) = 0$ is

$$\vartheta_k = \frac{1}{\zeta} \int_0^t e^{-[(Ck^4 + MH_\parallel k^2)(t-t')]/\zeta} f_k^\vartheta(t') dt',$$

which gives (y_k is the Fourier amplitude of the displacement)

$$\langle y_k y_{-k} \rangle = \frac{k_B T L}{Ck^4 + MH_\parallel k^2} \left[1 - \exp\left(- \frac{2(Ck^4 + MH_\parallel k^2)t}{\zeta} \right) \right].$$

As a result, applying $\Sigma_n \to (L/2\pi) \int dk$, the mean quadratic displacement of the filament can be expressed as follows:

$$\begin{aligned} \langle y^2(l,t) \rangle &= 2 \frac{k_B T}{2\pi} \int_0^\infty dk \frac{1}{(Ck^4 + MH_\parallel k^2)} \\ &\quad \times \left[1 - \exp\left(- \frac{2(Ck^4 + MH_\parallel k^2)t}{\zeta} \right) \right] \\ &= 2 \frac{k_B T t^{3/4}}{C^{1/4} \sqrt{2} (2\zeta)^{3/4}} \frac{1}{2\pi} \int_0^\infty \frac{dz[1 - \exp(-z)]}{z\sqrt{A^2 + z}\sqrt{\sqrt{A^2+z} - A}}. \end{aligned} \tag{5}$$

It depends on parameter $A = l_p / l_H^2 \sqrt{k_B T t / 2\zeta l_p}$, where $l_p = C/k_B T$ is the persistence length of the magnetic filament, and l_H is a magnetic healing length defined as $l_H = \sqrt{C/MH_\parallel}$. In the case when l_H is large or A small, the scaling law of the semiflexible polymers [15] $\langle y^2 \rangle \sim t^{3/4}$ is obtained. The scaling of the mean quadratic displacement as $t^{3/4}$ for the chain of superparamagnetic particles at moderate values of the magnetodipolar interaction parameter is illustrated in [17]. In the case when A is large, the scaling law $t^{1/2}$ follows.

For the analysis of the thermal fluctuations of the filaments in different situations, the formalism of the Green functions is useful. This allows one, for example, using the higher transcendental functions to obtain the expressions for different quantities in concise forms, which are convenient for the derivation of the asymptotic relations and scaling laws. Using the Green function G, the solution of the Cauchy problem with $y(l,0) = 0$ of the equation for the displacement of the filament, where the forces f of random thermal noise are accounted,

$$\partial_t \zeta y = - \partial_{llll} C y + MH_\parallel \partial_{ll} y + f(l,t), \tag{6}$$

reads

$$y(l,t) = \frac{1}{\zeta} \int_0^t \int_{-\infty}^\infty G(l-l', t-t') f(l',t') dl' dt', \tag{7}$$

G can be expressed by the Fourier integral

$$G(x,t) = \frac{1}{2\pi} \int_{-\infty}^\infty \exp[-(Ck^4 + MH_\parallel k^2)t/\zeta] \exp(ikx) dk \, \theta(t),$$

where θ is the Heaviside function. Accounting for the correlation function of the random thermal noise,

$$\langle f(x,t) f(x',t') \rangle = 2\zeta k_B T \delta(x-x') \delta(t-t'),$$

the mean quadratic displacement can be expressed as follows:

$$\langle y^2(l,t) \rangle = \frac{2k_B T}{\zeta} \int_0^t \int_{-\infty}^\infty G^2(l-l', t-\tau) d\tau dl'. \tag{8}$$

Using the identity

$$\int_0^\infty x^{\alpha-1} \exp(-px^2 - qx) dx = D_{-\alpha}(q/\sqrt{2p}) \Gamma(\alpha) \exp(q^2/8p) (2p)^{-\alpha/2},$$

the integral $\int_{-\infty}^\infty G^2(l-l',t) dl'$ in Eq. (8) can be expressed through the function of the parabolic cylinder $D_\nu(z)$ as follows:

$$\int_{-\infty}^{\infty} G^2(l-l',t)dl' = \frac{1}{\sqrt{8\pi}} D_{-1/2}\left(MH_\parallel \sqrt{\frac{t}{C\zeta}}\right) \times \exp[(MH_\parallel)^2 t/4C\zeta]\left(\frac{\zeta}{Ct}\right)^{1/4}. \quad (9)$$

Using asymptotics of the parabolic cylinder function $D_\nu(z)$ at $z\to\infty$,

$$D_{-1/2}(z) \sim z^{-1/2}\exp(-z^2/4),$$

for large $MH_\parallel\sqrt{t/C\zeta}$ we have

$$\int_{-\infty}^{\infty} G^2(l-l',t)dl' \sim \frac{1}{2\sqrt{2\pi}}\left(\frac{\zeta}{tMH_\parallel}\right)^{1/2}$$

and as a result

$$\langle y^2(l,t)\rangle \sim k_B T \sqrt{\frac{2}{\pi}}\left(\frac{t}{\zeta MH_\parallel}\right)^{1/2}.$$

This result, taking into account

$$\frac{1}{2\pi}\int_0^\infty \frac{[1-\exp(-z)]}{z^{3/2}}dz = \frac{1}{\sqrt{\pi}},$$

coincides with that given by the relation (5) for the large values of the parameter A. Asymptotics of the right side of the relation (9) for the small field gives

$$\int_{-\infty}^{\infty} G^2(l-l',t)dl' \sim \frac{\Gamma(1/4)}{4\pi}\left(\frac{\zeta}{2Ct}\right)^{1/4}$$

and

$$\langle y^2(l,t)\rangle \sim \frac{2^{3/4}}{3\pi}\frac{k_B T}{\zeta^{3/4}C^{1/4}}\Gamma(1/4)t^{3/4},$$

which coincides with that given by the relation (5) since

$$\int_0^\infty [1-\exp(-z)]z^{-7/4}dz = \frac{4\Gamma(1/4)}{3}.$$

The expression of the mean quadratic displacements of the magnetic filaments through the Green function allows one to establish the scaling laws for different situations of the filament deformation. Introducing $G_0(l,t)=\int_0^t G(l,\tau)d\tau$, we have

$$\int_{-\infty}^{\infty}\int_0^t G^2(l,t-\tau)dl d\tau = \frac{1}{2}G_0(0,2t),$$

and the displacement of the point of the application of the constant unit point force $\delta(x)$ at $t\geqslant 0$ is

$$y(0,t) = \frac{G_0(0,t)}{\zeta}.$$

The scaling laws for the displacement of the free and clamped end of the filament under the applied constant force at its end and the mean square displacement obey the same scaling laws. Indeed, since the solution of the mixed problem for the semi-infinite magnetic filament $l\in[-\infty,0]$ at the unit force applied to its right end by symmetry arguments is $2G_0(l,t)/\zeta$, then the scaling laws for the mean square displacement and the displacement of the free and clamped end under the action of the constant force are the same [15]. In [6], by the numerical simulation of the dynamics of the magnetic filament under the action of constant force applied at its free end, it is shown that the crossover from $t^{3/4}$ to $t^{1/2}$ at the increase of the magnetoelastic number $Cm=MHL^2/C$ takes place in this case also.

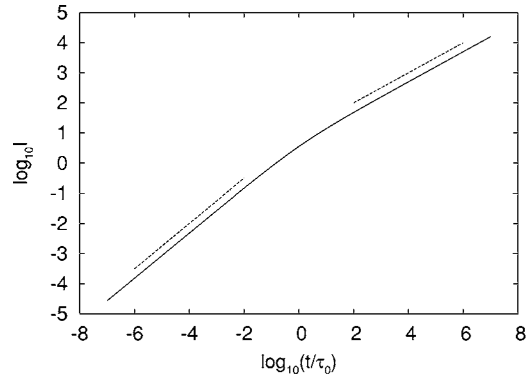

FIG. 2. The crossover of the time dependence of the mean quadratic displacement. Dashed lines correspond to the scalings $t^{3/4}$ and $t^{1/2}$ at small and large times, respectively.

The proportionality of the square root of the mean quadratic displacement $\sqrt{\langle y^2\rangle}$ to $t^{1/4}$ was found in [18] for the chains of superparamagnetic particles and is valid for several decades of the observation time. The crossover from $t^{3/4}$ to $t^{1/2}$ is illustrated by Fig. 2, where

$$I = A^{3/2}\int_0^\infty \frac{dz[1-\exp(-z)]}{z\sqrt{A^2+z}\sqrt{\sqrt{A^2+z}-A}}$$

in dependence on dimensionless time t/τ_0 is shown in logarithmic coordinates $[\tau_0=2\zeta l_H^4/k_B T l_p]$.

For the chain of single domain ferromagnetic particles, the parameter A can be expressed in terms of the parameters characterizing their properties. Since the persistence length in this case can be found as $l_p=\lambda d/2$, where d is a diameter of a particle [5], but $M=m/d$, where m is the magnetic moment of the particle, then $A=\xi/2\sqrt{\lambda}\sqrt{D_0 t/d^2}$, where $\lambda=m^2/d^3 k_B T$ is the parameter of the magnetodipolar interaction [19], $\xi=mH/k_B T$ is the Langevin parameter of the particles, and $D_0=k_B T/\zeta d$ is the characteristic value of their translation diffusion coefficient.

Scaling $D_0 t/d^2$ for the representation of the experimental data on the fluctuations of the chains of superparamagnetic particles at different values of the magnetodipolar interaction parameter is used in [20]. Nevertheless, the presented experimental data do not allow one to make definite conclusions about the crossover.

III. DYNAMIC MAGNETIC SUSCEPTIBILITY: CONTRIBUTION OF STRAIGHTENING OF THERMAL FLUCTUATIONS

Equation (4) allows one to calculate the contribution of the straightening of the thermal fluctuations of the semiflexible magnetic filaments to its dynamic magnetic susceptibility. In the case of the ac magnetic field, it reads

$$\zeta\frac{d\vartheta_k}{dt} = -Ck^4\vartheta_k - MH_{\parallel}k^2\cos\omega t\,\vartheta_k + f_k^{\vartheta}.$$

The transition to the stationary state is described by

$$\vartheta_k = \int_{-\infty}^{t} e^{-\int_{t'}^{t} f(t'')dr\, t''}\frac{1}{\zeta}f_k^{\vartheta}(t')dt',$$

where $f(t)=(Ck^4+MH_{\parallel}k^2\cos\omega t)/\zeta$.

The component of the magnetic moment of the filament in the direction of the applied field is

$$m = \int M\cos\vartheta dl. \tag{10}$$

In the case when the deviations from the straight shape of the filament oriented at the angle β with the respect to the field are small, we can expand the relation (10) with respect to $\tilde{\vartheta}$ ($\vartheta=\beta+\tilde{\vartheta}$, the tilde is omitted). Retaining only the first nonvanishing term, the relation (10) gives

$$m = \int\left(M\cos\beta - M\sin\beta\vartheta - \frac{M\cos\beta\vartheta^2}{2}\right)dl. \tag{11}$$

Since the suspension of the filaments has no spontaneous magnetization and the mean value of the tangent angle fluctuation ϑ is zero, only the last term in parentheses on the right side of the relation (11) contributes to the magnetization of the filament in an ac field.

Thus dynamic magnetic susceptibility is determined as

$$\chi = -\frac{1}{4}\int_0^{\pi} M\cos\beta\sin\beta\int\langle\vartheta^2(\beta)\rangle dld\beta/H. \tag{12}$$

Since $\langle\int\vartheta^2(\beta)dl\rangle=(1/L)\Sigma_k\langle\vartheta_k\vartheta_{-k}\rangle$ and up to the first-order terms in the amplitude of the external field

$$\begin{aligned}\frac{1}{L}\langle\vartheta_k\vartheta_{-k}\rangle = \frac{2k_BTk^2}{\zeta}\int_0^{\infty}\exp\left(-\frac{2Ck^4}{\zeta}\tau\right)\\ \times\left(1-\frac{2MH\cos\beta k^2}{\zeta\omega}\sin\omega t(1-\cos\omega\tau)\right.\\ \left.-\frac{2MH\cos\beta k^2}{\zeta\omega}\cos\omega t\sin\omega\tau\right)d\tau,\end{aligned} \tag{13}$$

then for the magnetic susceptibility from Eqs. (12) and (13) we have ($\tau_0=\zeta/2Ck^4$)

$$\begin{aligned}\chi = \frac{1}{3}\frac{M^2}{C^2}k_BTL\left(\sin(\omega t)\frac{1}{2\pi}\int_0^{\infty}\frac{\omega\tau_0 dk}{k^4[1+(\omega\tau_0)^2]}\right.\\ \left.+\cos(\omega t)\frac{1}{2\pi}\int_0^{\infty}\frac{dk}{k^4[1+(\omega\tau_0)^2]}\right).\end{aligned}$$

Introducing as the characteristic relaxation time $\tau_p=\zeta l_p^3/2k_BT$—the Brownian relaxation time of the rigid rod with the length equal to the persistence length for the real and imaginary parts of the complex susceptibility $\chi=\chi'-i\chi''$—we have

$$\chi' = \frac{1}{3}\frac{M^2l_p^3k_BTL}{C^2}\frac{1}{2\pi}\int_0^{\infty}\frac{k^4dk}{k^8+(\omega\tau_p)^2}, \tag{14}$$

$$\chi'' = \frac{1}{3}\frac{M^2l_p^3k_BTL}{C^2}\frac{1}{2\pi}\int_0^{\infty}\frac{\omega\tau_p dk}{k^8+(\omega\tau_p)^2}. \tag{15}$$

The prefactor in relations (14) and (15) represents the Langevin susceptibility $(1/3)[(Ml_p)^2/k_BT]$ of L/l_p independent magnetic elements. Relations (14) and (15) show that both real and imaginary parts of complex susceptibility obey the scaling law $(\omega\tau_p)^{-3/4}$ in dependence on the frequency.

We can obtain an equivalent expression by the formalism of the Green functions. According to Eq. (12), the transient filament magnetization for arbitrary value of the magnetic field at $\vartheta(l,0)=0$ can be expressed through the function of a parabolic cylinder as follows:

$$\begin{aligned}m = -\frac{1}{4}\int_0^{\pi} M\cos\beta\sin\beta d\beta\frac{k_BTL}{2^{1/2}\zeta^{1/4}C^{3/4}}\frac{1}{2\pi}\\ \times\int_0^t\frac{dy}{y^{3/4}}D_{-3/2}[q(t,y)/\sqrt{2}]\Gamma(3/2)\exp[q^2(t,y)/8],\end{aligned}$$

where

$$\begin{aligned}q(t,y) = \frac{\sqrt{2}MH\cos\beta}{\sqrt{\zeta C}\omega}\left(\frac{\sin(\omega t)[1-\cos(\omega y)]}{\sqrt{y}}\right.\\ \left.+\frac{\cos(\omega t)\sin(\omega y)}{\sqrt{y}}\right).\end{aligned}$$

If $\tau/\tau_p \gg 1/\sqrt{\omega\tau_p}$ ($\tau=\zeta l_p^2/MH$ is the characteristic magnetic relaxation of the rod with the length equal to the persistence length), then for the average magnetization in the transitory stage we have

$$\begin{aligned}m = \frac{1}{3}\frac{\Gamma(5/4)}{2^{5/4}}\frac{k_BTLM^2H}{(\omega\zeta)^{3/4}C^{5/4}}\frac{1}{2\pi}\left(\sin(\omega t)\int_0^{\omega t}\frac{(1-\cos z)}{z^{5/4}}dz\right.\\ \left.+\cos(\omega t)\int_0^{\omega t}\frac{\sin z}{z^{5/4}}dz\right).\end{aligned} \tag{16}$$

At $\omega t \gg 1$, for the real and imaginary parts of the complex susceptibility from Eq. (16) we obtain

$$\chi' = \frac{1}{3}\frac{\Gamma(5/4)}{2^{5/4}}\frac{k_BTLM^2}{(\omega\zeta)^{3/4}C^{5/4}}\frac{1}{2\pi}\int_0^{\infty}\frac{\sin z}{z^{5/4}}dz$$

and

$$\chi'' = \frac{1}{3}\frac{\Gamma(5/4)}{2^{5/4}}\frac{k_BTLM^2}{(\omega\zeta)^{3/4}C^{5/4}}\frac{1}{2\pi}\int_0^{\infty}\frac{(1-\cos z)}{z^{5/4}}dz.$$

Since

$$\int_0^\infty \frac{dx}{1+x^8} = \frac{\Gamma(5/4)}{4}\int_0^\infty \frac{(1-\cos z)}{z^{5/4}}dz$$

and

$$\int_0^\infty \frac{x^4 dx}{1+x^8} = \frac{\Gamma(5/4)}{4}\int_0^\infty \frac{\sin z}{z^{5/4}}dz,$$

these expressions of the real and imaginary parts of the complex susceptibility are equivalent to that given by the relations (14) and (15).

The results obtained in Secs. II and III correspond to the case of enough small times or large frequencies when the characteristic deformation penetration length remains less than the length of filament. At large times the mean quadratic displacement of the filament saturates to the constant value determined by the equipartition theorem. An illustration that in the case of filaments with finite length the dynamic magnetic susceptibility obeys at the high frequencies the scaling law $\omega^{-3/4}$ is given in the next part of the work.

IV. THE DYNAMIC MAGNETIC SUSCEPTIBILITY OF THE FILAMENT WITH A FINITE LENGTH

For the filament with finite length, the contribution to its magnetization is coming also from the bending of its free ends. In the case when the thermal fluctuations are absent up to the first-order terms in the applied field, the displacement of the filament, making angle β with the direction of the ac magnetic field, as it follows from Eq. (6), obeys the equation

$$\partial_t \zeta y = -\partial_{llll} C y. \tag{17}$$

In this case, the magnetic field enters only through the boundary conditions, which applied at the ends of the filament $l=0$ and $l=L$ correspond to the absence of momentum stress

$$\partial_{ll} y = 0 \tag{18}$$

and vanishing of the total normal force

$$-\partial_{lll} C y + MH \sin\beta = 0. \tag{19}$$

We see that the problem is equivalent to the problem of the bending of the Kirchhoff rod under the action of the normal force applied at its ends, which is given up to the linear in the field terms by $MH\sin\beta$.

Looking for the solution of Eq. (17) at boundary conditions (18) and (19) in the form $y=\tilde{y}e^{-\lambda t}$ (the tilde is omitted), we obtain

$$\zeta\lambda y = \partial_{llll} C y.$$

Scaling the arclength l with respect to L, but λ with respect to $C/\zeta L^4$ ($\tilde{\lambda}=\zeta L^4\lambda/C$, the tilde is omitted), the following eigenvalue problem is obtained for the case of the homogeneous boundary conditions

$$\lambda y = \partial_{llll} y,$$

$$\partial_{ll} y|_{0,1} = 0,$$

$$\partial_{lll} y|_{0,1} = 0.$$

The eigenfunctions are ($p=\lambda^{1/4}$)

$$\varphi_k = [\cosh(p_k l) + \cos p_k l][\sinh p_k - \sin p_k] - [\cosh(p_k) - \cos p_k][\sinh p_k l + \sin p_k l],$$

where p_k are found from the equation

$$\cosh(p_k)\cos p_k = 1.$$

The eigenvalue $\lambda=0$ is degenerate and corresponds to the two orthogonal eigenfunctions $y_0=1$ and $y_1=(l-1/2)$. The particular solution orthogonal to the eigenfunctions with $\lambda=0$ of the problem at the nonhomogeneous boundary conditions in the field $H(t)$ is $[h(t)=M\sin(\beta)H(t)L^3/C]$

$$y = h(t)\frac{1}{420}(42l^5 - 105l^4 + 70l^3 - 9l + 1) = h(t)f(l).$$

Solution of the Cauchy problem for Eq. (17) at boundary conditions (18) and (19) is as follows:

$$y = \sum_k \frac{\lambda_k f_k \varphi_k}{a_k^2}\int_{-\infty}^t e^{-\lambda_k(t-t')}h(t')dt' + y_0(t)(l-1/2),$$

where y_0 obeys the differential equation

$$\frac{dy_0}{dt} = -12h(t)$$

and describes the rotation of the magnetic filament as the rigid rod, but $f(l)=\Sigma_k f_k\varphi_k(l)/a_k^2; a_k^2=\int_0^1\varphi_k^2 dl$. These relations allow one to calculate the complex magnetic susceptibility under the action of an ac field $H(t)=H_0\exp(i\omega t)$. Magnetic susceptibility due to rotation of the filament as a rigid rod accounting for its rotational Brownian motion is

$$\frac{(ML)^2}{3k_BT}\frac{1}{1+i\tau_B\omega},$$

where $\tau_B=\alpha/2k_BT$ and the rotational drag coefficient α is found according to the relation $\alpha=\zeta L^3/12$. According to the relation (11), the magnetic moment of the filament is

$$m = -M\sin\beta[y(1,t) - y(0,t)]. \tag{20}$$

Relation (20) after averaging over the random orientations of the filaments for the complex magnetic susceptibility gives ($\tau_k=\lambda_k^{-1}$)

$$\chi = -\frac{M^2L^3}{3C}\sum_k \frac{f_k(\varphi_k(1)-\varphi_k(0))}{a_k^2(1+i\omega\tau_k)}.$$

We see that due to the heterogeneous deformation of the filament, there is the distribution of the relaxation times τ_k determined by the eigenvalues of the eigenmodes λ_k. The second and fourth normalized odd eigenmodes $\varphi_{2,4}$ in dependence on the contour length of the filament are shown in Fig. 3. Contribution of the other odd modes is small. Even modes do not contribute to the filament magnetization in this case at all. As we see from Fig. 3, the most significant contribution to the magnetization comes from the second odd mode and corresponds to the periodic turning of the free ends of the

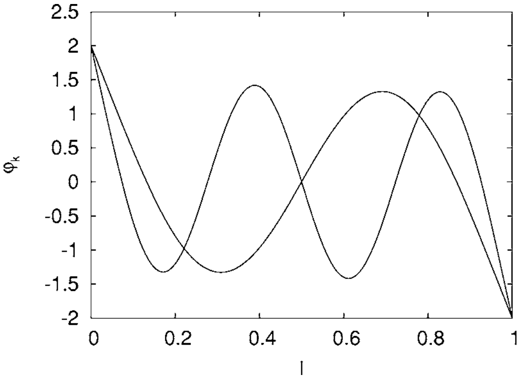

FIG. 3. The first two odd eigenmodes of the filament deformation.

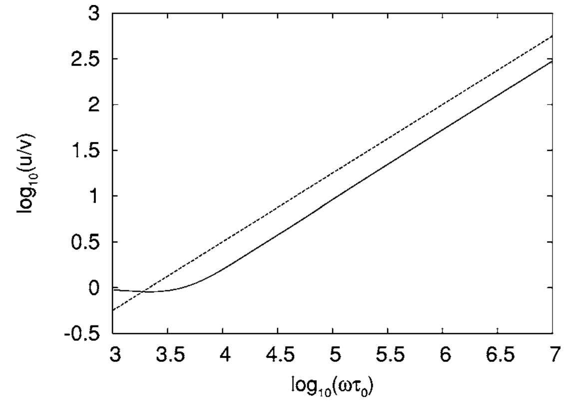

FIG. 4. Transition to the scaling law for the magnetic susceptibility. Dashed line corresponds to $\omega^{-3/4}$ dependence of the magnetic susceptibility.

filament in the direction of an ac magnetic field.

A more convenient form for the investigation of the scaling laws arising due to this distribution is the closed form of the solution for the displacement of the filament at the excitation of the deformation modes under the action of an ac field. For the Fourier component $\Delta y(\omega)$ of the difference between the displacements of the ends of the filament $y(1,t)-y(0,t)$, gives $[k=(\omega\zeta L^4/C)^{1/4}\exp(-i\pi/8)]$

$$\Delta y(\omega)=\frac{MHL^3\sin\beta}{C}\frac{2[(\cosh k-1)\sin k+(1-\cos k)\sinh k]}{k^3(1-\cosh k\cos k)}. \tag{21}$$

The asymptotics of the relation (21) for the magnetic susceptibility after averaging over random orientations of the filament gives

$$\chi=-\frac{M^2L^3}{3C}2\sqrt{2}\left(\frac{C}{\omega\zeta L^4}\right)^{3/4}\exp(i5\pi/8).$$

We see that the scaling law $\omega^{-3/4}$ for the real and imaginary parts of the complex magnetic susceptibility is valid at large frequencies of the applied field.

The relations (20) and (21) show that the magnetic susceptibility of the single filament is proportional to $\sin^2\beta$. This and the positive sign of the real part of the complex magnetic susceptibility show that the flexible magnetic filament in the magnetic field of the high frequency should orientate in the direction perpendicular to the field. Study of this interesting effect is planned for future publications.

Distribution of the magnetic relaxation times of the magnetic colloids is usually analyzed by Cole-Cole plots. In the case when the magnetic susceptibility has the form [21]

$$\chi=\chi_\infty+\frac{\chi_0-\chi_\infty}{1+(i\omega\tau_0)^{1-\alpha}}, \tag{22}$$

the Cole-Cole plot is the arc of a circle that cross-sections with the real axis determined by the static and high-frequency susceptibilities χ_0 and χ_∞, respectively. Exponent α characterizes the distribution of the relaxation times. It is determined from the plot of ratio $u/v=\sqrt{(\chi'-\chi_0)^2+\chi''^2}/\sqrt{(\chi'-\chi_\infty)^2+\chi''^2}$, which, when the relation (22) is true, is equal to $(\omega\tau_0)^{1-\alpha}$, in dependence on the frequency of the ac field [21]. According to our model, $\chi_0=M^2L^3/630C$, but $\chi_\infty=0$. The plot of u/v in Fig. 4 shows that crossover to scaling law $\chi\sim(\omega\tau_0)^{-3/4}$ takes place at $\omega\tau_0\simeq 10^4$. Such values are reasonable for magnetic colloids containing chainlike aggregates of ferromagnetic particles, such as those investigated in [8]. A simple estimate of $\omega\zeta L^4/C$ using the value of the curvature elasticity constant $m^2/2d^2$ [5], where m is the magnetic moment of the particle but d its diameter, shows that $\omega\tau_0\simeq 10^4$ for the chain containing 30 Fe particles is reached at the frequency of the ac field 6.1 kHz (ζ is estimated as $4\pi\eta$, where for η the viscosity of water is taken, but $m=\pi d^3M/6$, where $M=1700$ G is the saturation magnetization of Fe). This estimate shows that the experimental observation of the scaling law $\omega^{-3/4}$ is realistic. Indeed, the experimental data for Fe colloid with the particle radius 8.2 nm [8], for which the magnetodipolar interaction parameter $\lambda=m^2/d^3k_BT$ is huge—approximately 90—and thus contains long chains of the ferromagnetic particles, show behavior close to that predicted by our model. The experimental data of [8] together with the expected scaling law for the magnetic colloid containing chainlike aggregates are shown in Fig. 5. We observe the transition to the scaling behavior at the frequency of the ac field of about 10 kHz, which is very close to our estimate. It should be

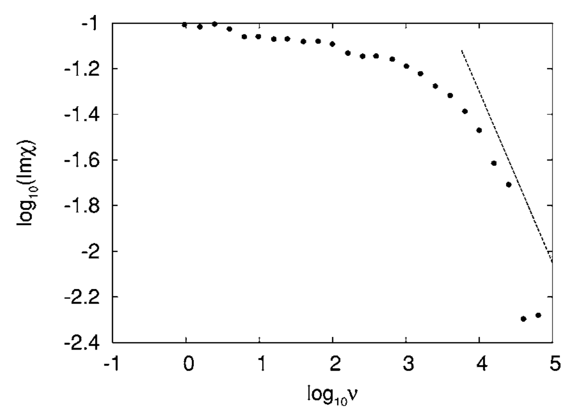

FIG. 5. Experimental data [8] for the frequency dependence of the imaginary part of the complex magnetic susceptibility. By dashed line the power law $\chi''\sim\omega^{-3/4}$ is shown.

noted that $\omega\tau_0$ due to the scalings considered does not depend on the diameter of particles and scales with the number of particles in the chain N as N^4. More experimental data for the magnetic susceptibility in the high-frequency range, possibly by using the viscous carrier liquids, are highly desirable.

V. CONCLUSIONS

The characteristic scaling law of the time dependence of the mean quadratic displacement of the magnetic filaments under the action of enough strong magnetic field shows the crossover from $t^{3/4}$ characteristic of the semiflexible filaments to $t^{1/2}$ characteristic of the magnetic filaments. The general formalism of the Green functions allows one to show that this crossover takes place at different regimes of the deformation of the magnetic filaments. It is shown that the deformation modes of the magnetic filaments lead to the distribution of the magnetization relaxation times. It leads in the high-frequency case to the scaling law of the magnetic susceptibility $\omega^{-3/4}$. This scaling law is valid both in the case of the straightening of the thermal fluctuations of the filaments in the applied field and bending of their free ends due to the torques of the applied field.

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